COMC

december 2012

1 uitleg

De COMC is een internationale wedstrijd met een 10 tal deelnemende landen.
De wedstrijd duurt 4 uur waarin 4 opgaven mogen worden opgelost.

Er zijn 2 categorieen:

"This division is made mostly because there are international competitions for younger students such as (MEMO) and (JBMO)
so the first category usually assumes less mathematical knowledge (so no functional equations polynomials etc.)
while the second category is supposed to be IMO-like."

1. Er is de categorie gelijkaardig aan de VWO, waarbij de onderwerpen uitgebreid zijn.
   `http://www.webklik.nl/user_files/2011_09/307951/stellingenvoorbeelden-Bundel.pdf` geeft uitleg over deze onderwerpen met voorbeelden.
   Hier zijn de basis van de problem-solvingtechnieken zeker handig.

2. De JWO-categorie zal logischer moeten bewijzen. (enkel basis zoals invarianten, het ongerijmde, inductie waarschijnlijk)

De datum kan met lichte afwijking gekozen worden, maar moet tussen 1 en 8 december liggen. In die periode is het verboden de problemen online te publiceren! (zeker niet wereldwijd)
De normale datum was 1/12/12 van 9 tot 13 uur.

Een vijftal minuten op voorhand worden de vragen doorgezonden in het Engels en het Nederlands in PDF.

Voor de Vlamingen geeft de Croatische organisatie 2 mogelijkheden gegeven:

"

5.1 You can ask your students to write in English (it is really not important to have grammar punctuality) and we allow them to use any dictionaries or other English related help if they need it. In this case you can help mark their work it would be really helpful but you don't need to translate any of their work.

    Dit betekent dat jullie alles kort in het Engels schrijven.

5.2 You can allow them to write in Flemish. In this case we will have to ask you to translate and mark their work and send it again (so we would like to receive Flemish originals no later than 4 and a half hour after the designated start of the competition) but we would like to recieve the translated documents in a week after the day of the competition. You are not asked to literally translate students work or retype all their equations but simply to in short lines explain what is the student doing and comment any given mark(even if you have not awarded any marks please do translate any ideas student had just in case). You will recieve a detailed marking scheme to work on.

"Het werk zou ingescand of in PDF doorgegeven moeten worden.
Hiervoor zal een beetje tijd meegegeven worden na de 13 uur volgens hoe men werkt.

Als voorbeeld geven we de eerste editie mee met een voorbeeld van een korte oplossing in het Engels.
(door elegant in punten de oplossing te schetsen kan het sneller worden doorgegeven)

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1^{st} BALKAN STUDENT MATHEMATICAL COMPETITION

1. Matematičko natjecanje učenika Balkana

November 2008.

3^{rd} and 4^{th} grade

Problem 1. Find all functions $f\colon \mathbb{R} \to \mathbb{R}$ such that for every two real numbers x and y,

$$f\left(f\left(x\right) + xy\right) = f\left(x\right) \cdot f\left(y+1\right).$$

(Marko Radovanović)

Problem 2. Paralampius the Gnu stands on number 1 on number line. He wants to come to a natural number k by a sequence of consecutive jumps. Let us denote the number of ways on which Paralampius can come from number 1 to number k with $f\left(k\right)$ ($f\colon \mathbb{N} \to \mathbb{N}_0$). Specially, $f\left(1\right) = 0$. A way is a sequence of numbers (with order) which Paralampius has visited on his travel from number 1 to number k. Paralampius can, from number b, jump to number

- $2b$ (always),
- $3b$ (always),
- b^2 $\left(\text{if } \dfrac{b^4}{6k} \in \mathbb{N}, \text{ where } k \text{ is a natural number on which he wants to come to in the end}\right)$.

Prove that, for every natural number n, there exists a natural number m_0 such that for every natural number $m > m_0$,

$$f\left(m\right) < 2^{\alpha_1+\alpha_2+\ldots+\alpha_i-n},$$

where $m = p_1^{\alpha_1} \cdot p_2^{\alpha_2} \cdot \ldots \cdot p_i^{\alpha_i}$ ($p_1 < p_2 < \ldots < p_i$ are prime divisors of number m and i, α_1, α_2, $\ldots$, α_i are natural numbers) is a prime factorization of natural number m. It is known that this factorization is unique for every natural number $m > 1$.

(Melkior Ornik, Ivan Krijan)

Problem 3. A convex n-gon ($n \in \mathbb{N}$, $n > 2$) is given in the plane. Its area is less than 1. For each point X of this plane, we shall denote with $F\left(X\right)$ the area of the convex hull of point X and a given n-gon (the area of the minimal convex polygon which includes both the point X and a given n-gon). Prove that the set of points for which $F\left(X\right) = 1$ is a convex polygon with $2n$ sides or less.

Problem 4. Prove that for every natural number k, there exists infinitely many natural numbers n such that

$$\frac{n - d\left(n^r\right)}{r} \in \mathbb{Z}, \text{ for every } r \in \left\{1, 2, \ldots, k\right\}.$$

Here, $d\left(x\right)$ denotes the number of natural divisors of a natural number x, including 1 and x itself.

(Melkior Ornik)

Time allowed: 240 minutes.
Each problem is worth 10 points.
Write each problem on a separate paper.
Calculators or any other helping items, excluding rulers and compasses, are not allowed.

1.1 solution problem 1

We write $P(x, y)$ when we fill in x, y in the equation

$$f(f(x) + xy) = f(x) \cdot f(y + 1)$$

$P(0, y)$ gives $f(f(o)) = f(0)f(y + 1) = f(0)^2$ by $P(0, -1)$.

Hence $f(0) = 0$ or $f(y + 1) = f(0) = c$, then f is constant and $c = c^2$ so $c = 0$ or 1.

If there exist $t \neq 0 : f(t) = 0$
$P(t, y) : f(ty) = 0$ which means $f(x) = 0$ for all $x \in \mathbb{R}$.

Hence $f(t) = 0$ means $t = 0$ so

$P(x, -1)$ gives $f(f(x) - x) = 0$ means $f(x) = x$.

We conclude that $f(x) = 1, f(x) = 0, f(x) = x$ are the only three functions who satisfy the equation.

Het uittypen kan soms echter langer duren (zoals probleem 2), zodat het zelfs dan nog eventueel te lang kan duren om alles in het Engels in te dienen.
In dat geval geven we 5.2 door en lossen het zo op.